

A stylized, colorful illustration of a landscape. The foreground features rolling green hills with a brown path. On the left, there are several plants: a green tree, a purple flower, and an orange flower. A red and white butterfly is flying above the green tree. The background consists of wavy blue and white bands, suggesting a sky or water. The overall style is flat and modern.

# Revenue, Cost, and Profit

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## Topik utama

- *Mencari nilai profit dengan fungsi kuadrat*

## Fungsi Profit

$$\pi = TR - TC$$

The *total revenue* received from the sale of  $Q$  goods at price  $P$  is given by

$$TR = PQ$$

For example, if the price of each good is \$70 and the firm sells 300 then the revenue is

$$\$70 \times 300 = \$21\,000$$

## Example

Given the demand function

$$P = 100 - 2Q$$

express TR as a function of  $Q$  and hence sketch its graph.

- (a) For what values of  $Q$  is TR zero?
- (b) What is the maximum value of TR?

## Solution

Total revenue is defined by

$$TR = PQ$$

and, since  $P = 100 - 2Q$ , we have

$$TR = (100 - 2Q)Q = 100Q - 2Q^2$$

### Step 1

The coefficient of  $Q^2$  is negative, so the graph has an inverted U shape.

### Step 2

The constant term is zero, so the graph crosses the TR axis at the origin.

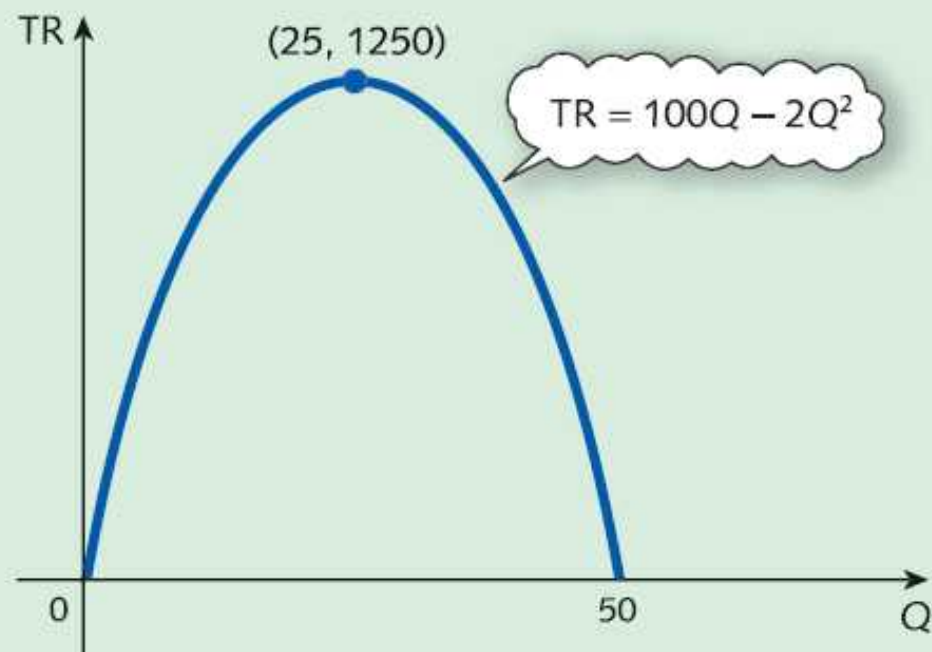
### Step 3

To find where the curve crosses the horizontal axis, we could use 'the formula'. However, this is not necessary, since it follows immediately from the factorization

$$TR = (100 - 2Q)Q$$

that  $TR = 0$  when either  $100 - 2Q = 0$  or  $Q = 0$ . In other words, the quadratic equation has two solutions,  $Q = 0$  and  $Q = 50$ .

Figure 2.6



- From Figure 2.6 the total revenue is zero when  $Q = 0$  and  $Q = 50$ .
- By symmetry, the parabola reaches its maximum halfway between 0 and 50, that is at  $Q = 25$ . The corresponding total revenue is given by

$$TR = 100(25) - 2(25)^2 = 1250$$



# Soal Latihan 1

1 Given the demand function

$$P = 1000 - Q$$

express TR as a function of  $Q$  and hence sketch a graph of TR against  $Q$ . What value of  $Q$  maximizes total revenue and what is the corresponding price?

In general, given the linear demand function

$$P = aQ + b \quad (a < 0, b > 0)$$

the total revenue function is

$$\begin{aligned} \text{TR} &= PQ \\ &= (aQ + b)Q \\ &= aQ^2 + bQ \end{aligned}$$

This function is quadratic in  $Q$  and, since  $a < 0$ , the TR curve has an inverted U shape. Moreover, since the constant term is zero, the curve always intersects the vertical axis at the origin. This fact should come as no surprise to you; if no goods are sold, the revenue must be zero.

## Total Cost

We now turn our attention to the *total cost* function, TC, which relates the production costs to the level of output,  $Q$ . As the quantity produced rises, the corresponding cost also rises, so the TC function is increasing. However, in the short run, some of these costs are fixed. *Fixed costs*, FC, include the cost of land, equipment, rent and possibly skilled labour. Obviously, in the long run all costs are variable, but these particular costs take time to vary, so can be thought of as fixed in the short run. *Variable costs*, on the other hand, vary with output and include the cost of raw materials, components, energy and unskilled labour. If VC denotes the variable cost per unit of output then the total variable cost, TVC, in producing  $Q$  goods is given by

$$TVC = (VC)Q$$

The total cost is the sum of the contributions from the fixed and variable costs, so is given by

$$TC = FC + (VC)Q$$

In general, the *average cost* function, AC, is obtained by dividing the total cost by output, so that

$$\begin{aligned} AC &= \frac{TC}{Q} \\ &= \frac{FC + (VC)Q}{Q} \\ &= \frac{FC}{Q} + \frac{(VC)Q}{Q} \\ &= \frac{FC}{Q} + VC \end{aligned}$$

## Example

Given that fixed costs are 1000 and that variable costs are 4 per unit, express TC and AC as functions of  $Q$ . Hence sketch their graphs.

## Solution

We are given that  $FC = 1000$  and  $VC = 4$ , so

$$TC = 1000 + 4Q$$

and

$$AC = \frac{TC}{Q} = \frac{1000 + 4Q}{Q} = \frac{1000}{Q} + 4$$

The graph of the total cost function is easily sketched. It is a straight line with intercept 1000 and slope 4. It is sketched in Figure 2.7. The average cost function is of a form that we have not met before, so we have no prior knowledge about its basic shape. Under these circumstances it is useful to tabulate the function. The tabulated values are then plotted on graph paper and a smooth curve obtained by joining the points together. One particular table of function values is

$Q$	100	250	500	1000	2000
$AC$	14	8	6	5	4.5

A graph of the average cost function, based on this table, is sketched in Figure 2.8. This curve is known as a *rectangular hyperbola* and is sometimes referred to by economists as being *L-shaped*.

Figure 2.7

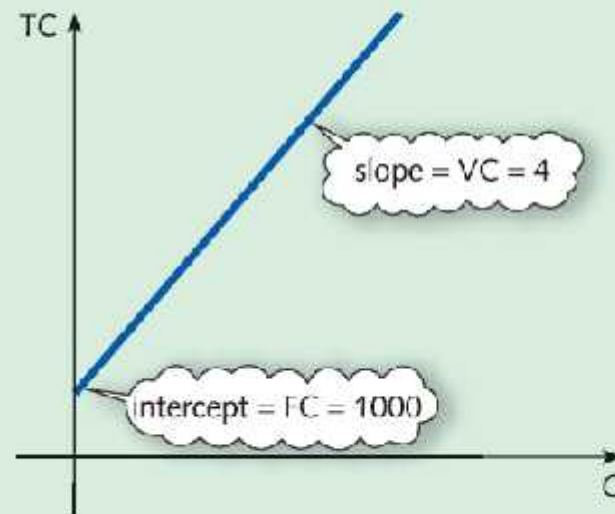
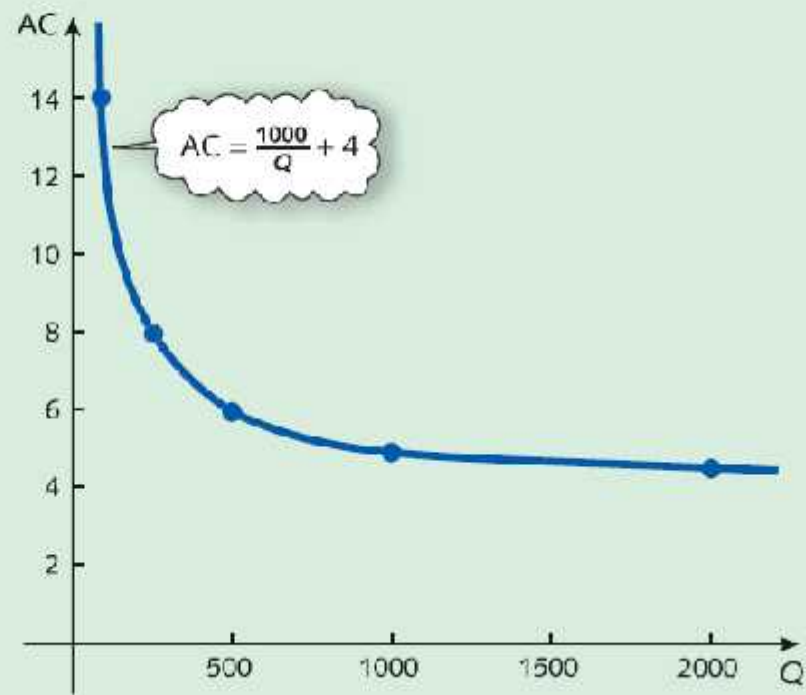


Figure 2.8



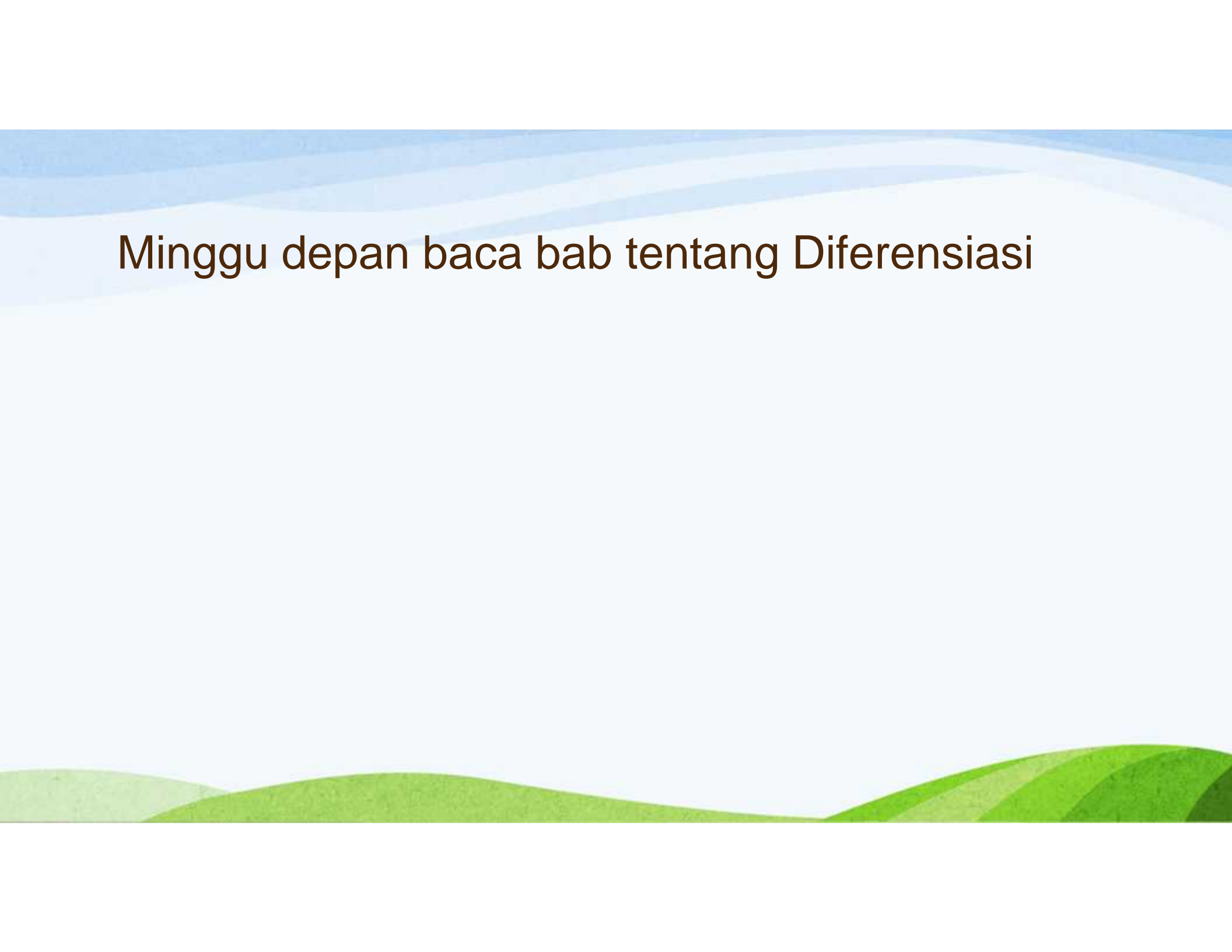


# Latihan soal 2

Given that fixed costs are 100 and that variable costs are 2 per unit, express TC and AC as functions of  $Q$ . Hence sketch their graphs.

The background features a series of horizontal, wavy bands in shades of blue and green, creating a sense of depth and movement. The top portion is dominated by various tones of blue, while the bottom portion transitions into shades of green. The overall effect is a soft, abstract landscape.

**Stop**



Minggu depan baca bab tentang Diferensiasi