

Supply and Demand Analysis

MATEMATIKA EKONOMI LANJUTAN
AL MUIZZUDDIN F

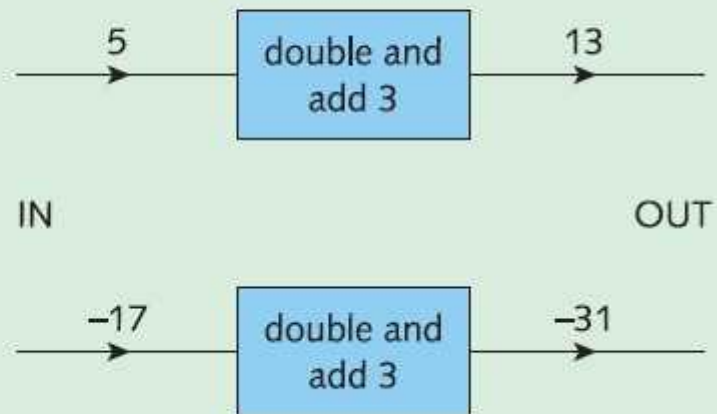


Fungsi

2

A *function*, f , is a rule which assigns to each incoming number, x , a uniquely defined outgoing number, y . A function may be thought of as a 'black box' that performs a dedicated arithmetic calculation. As an example, consider the rule 'double and add 3'. The effect of this rule on two specific incoming numbers, 5 and -17 , is illustrated in Figure 1.13 (overleaf).

Figure 1.13



Fungsi (2)

3

- Secara sederhana kita dapat menuliskan sebagai berikut..

$$y = 2x + 3 \quad \text{or} \quad f(x) = 2x + 3$$

Contoh 1

4

Example

(a) If $f(x) = 2x^2 - 3x$ find the value of $f(5)$.

(b) If $g(Q) = \frac{3}{5 + 2Q}$ find the value of $g(2)$.

Jawaban 1

5

Solution

(a) Substituting $x = 5$ into $2x^2 - 3x$ gives

$$\begin{aligned} f(5) &= 2 \times 5^2 - 3 \times 5 \\ &= 2 \times 25 - 3 \times 5 \\ &= 50 - 15 = 35 \end{aligned}$$

BIDMAS

(b) Although the letter Q is used instead of x , the procedure is the same.

$$g(2) = \frac{3}{5 + 2 \times 2} = \frac{3}{9} = \frac{1}{3}$$

Latihan Soal 1

6

Practice Problem

1 Evaluate

(a) $f(25)$

(b) $f(1)$

(c) $f(17)$

(d) $g(0)$

(e) $g(48)$

(f) $g(16)$

for the two functions

$$f(x) = -2x + 50$$

$$g(x) = -\frac{1}{2}x + 25$$

Do you notice any connection between f and g ?

Variabel : dependent dan independent

7

The incoming and outgoing variables are referred to as the *independent* and *dependent* variables respectively. The value of y clearly 'depends' on the actual value of x that is fed into the function. For example, in microeconomics the quantity demanded, Q , of a good depends on the market price, P . We might express this as

$$Q = f(P)$$



$$Q = f(P)$$

Such a function is called a *demand* function.

Penulisan Fungsi ke dalam persamaan Linear

8

Written in the form $P = g(Q)$, the demand function tells us that P is a function of Q but it gives us no information about the precise relationship between these two variables. To find this we need to know the form of the function which can be obtained either from economic theory or from empirical evidence. For the moment we hypothesize that the function is linear so that

$$P = aQ + b$$

$a = \text{parameter}$

$b = \text{parameter}$

Demand / Permintaan

9

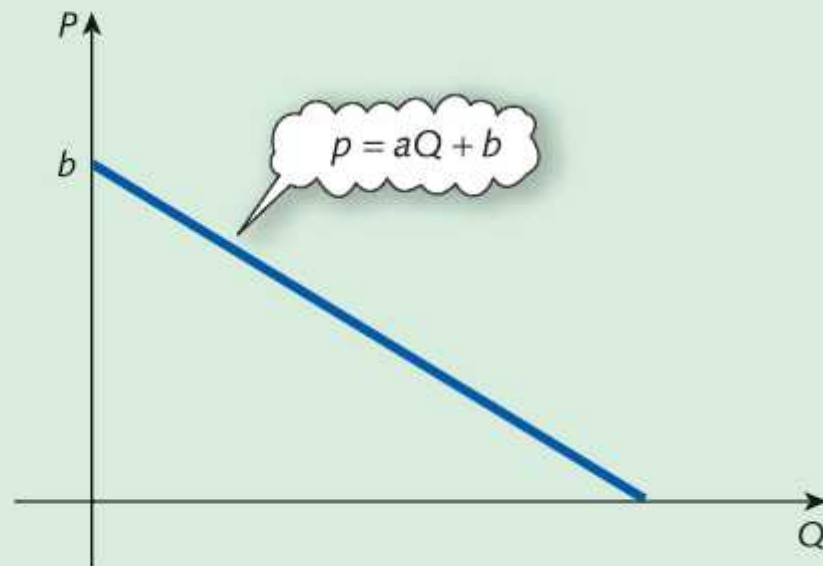


6/28/2016

Kurva Persamaan Demand

10

Figure 1.14



Lanjut

11

In symbols we write

$$a < 0$$

read 'a is
less than zero'

It is also apparent from the graph that the intercept, b , is positive: that is,

$$b > 0$$

read 'b is
greater than zero'

Contoh 2

12

Example

Sketch a graph of the demand function

$$P = -2Q + 50$$

Hence, or otherwise, determine the value of

- (a) P when $Q = 9$
- (b) Q when $P = 10$

Jawab 2

13

Solution

For the demand function

$$P = -2Q + 50$$

$a = -2$, $b = 50$, so the line has a slope of -2 and an intercept of 50 . For every 1 unit along, the line goes down by 2 units, so it must cross the horizontal axis when $Q = 25$. (Alternatively, note that when $P = 0$ the equation reads $0 = -2Q + 50$, with solution $Q = 25$.) The graph is sketched in Figure 1.15.

- (a) Given any quantity, Q , it is straightforward to use the graph to find the corresponding price, P . A line is drawn vertically upwards until it intersects the demand curve and the value of P is read off from the vertical axis. From Figure 1.15, when $Q = 9$ we see that $P = 32$. This can also be found by substituting $Q = 9$ directly into the demand function to get

$$P = -2(9) + 50 = 32$$

Lanjut

14

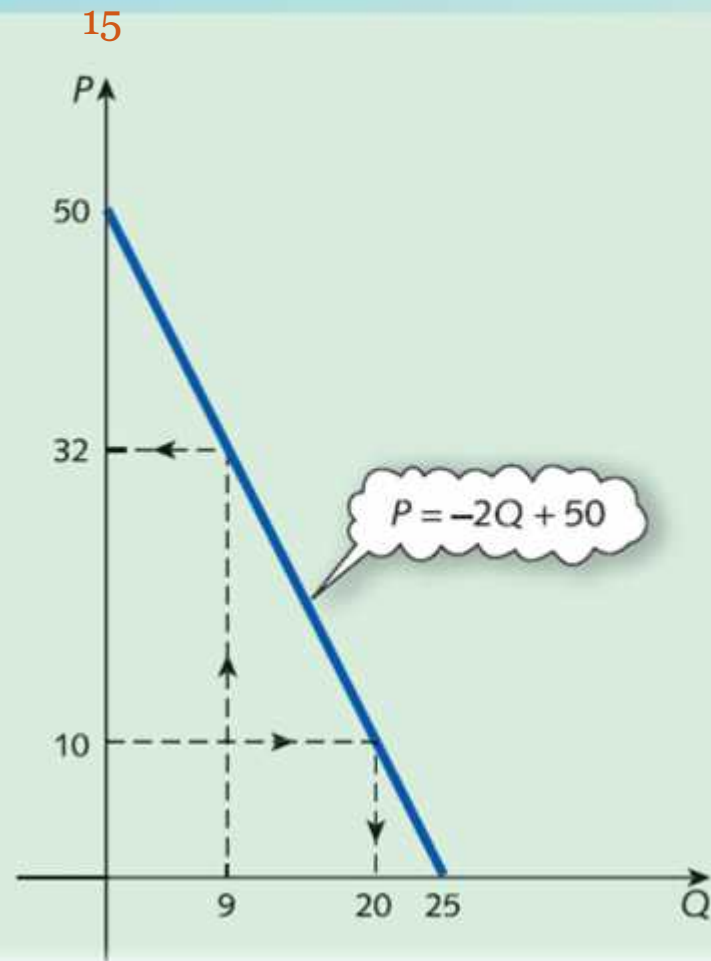
- (b) Reversing this process enables us to calculate Q from a given value of P . A line is drawn horizontally until it intersects the demand curve and the value of Q is read off from the horizontal axis. Figure 1.15 indicates that $Q = 20$ when $P = 10$. Again this can be found by calculation. If $P = 10$ then the equation reads

$$10 = -2Q + 50$$

$$-40 = -2Q \quad (\text{subtract } 50 \text{ from both sides})$$

$$20 = Q \quad (\text{divide both sides by } -2)$$

Figure 1.15



- **Ada Pertanyaan?**

It's easy..!

Soal Latihan 2

17

Practice Problem

2 Sketch a graph of the demand function

$$P = -3Q + 75$$

Hence, or otherwise, determine the value of

(a) P when $Q = 23$

(b) Q when $P = 18$

Supply / Penawaran

18



6/28/2016

Supply? What is?

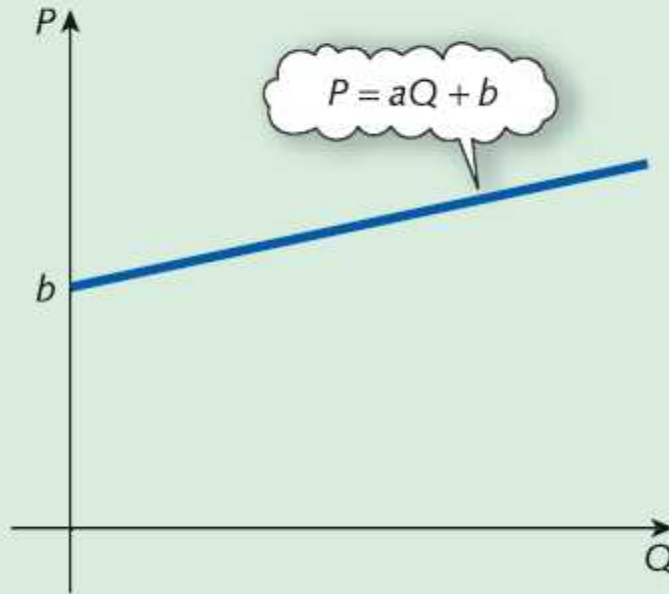
19

The *supply* function is the relation between the quantity, Q , of a good that producers plan to bring to the market and the price, P , of the good. A typical linear supply curve is indicated in Figure 1.18. Economic theory indicates that, as the price rises, so does the supply. Mathematically, P is then said to be an *increasing* function of Q . A price increase encourages existing producers to raise output and entices new firms to enter the market. The line shown in Figure 1.18 has equation

Kurva supply

20

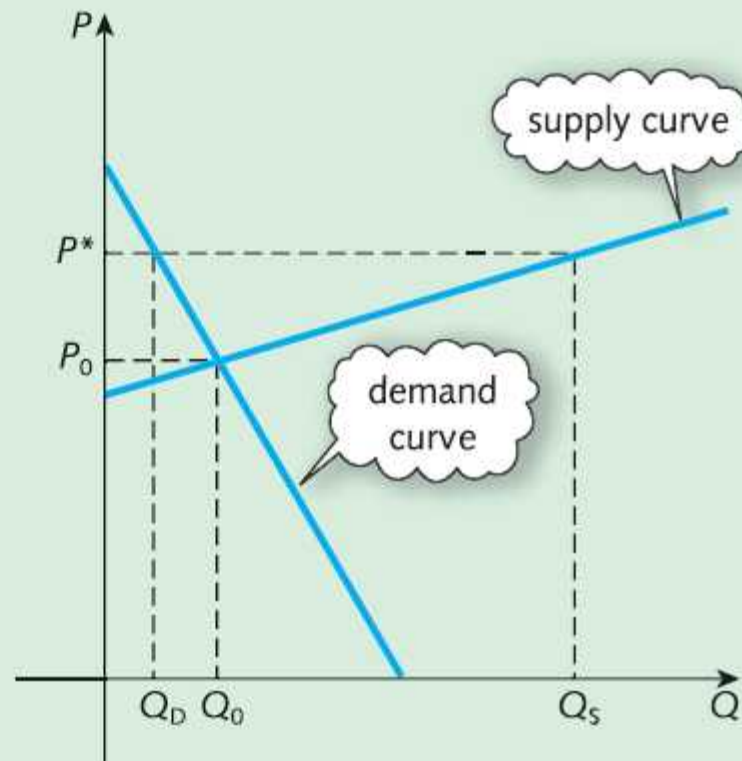
Figure 1.18



Keseimbangan supply dan demand

21

Figure 1.19



Contoh 3

22

Example

The demand and supply functions of a good are given by

$$P = -2Q_D + 50$$

$$P = \frac{1}{2}Q_S + 25$$

where P , Q_D and Q_S denote the price, quantity demanded and quantity supplied respectively.

- (a) Determine the equilibrium price and quantity.
- (b) Determine the effect on the market equilibrium if the government decides to impose a fixed tax of \$5 on each good.

Solution

Jawab 3

23

Solution

(a) The demand curve has already been sketched in Figure 1.15. For the supply function

$$P = \frac{1}{2}Q_s + 25$$

we have $a = \frac{1}{2}$, $b = 25$, so the line has a slope of $\frac{1}{2}$ and an intercept of 25. It therefore passes through $(0, 25)$. For a second point, let us choose $Q_s = 20$, say. The corresponding value of P is

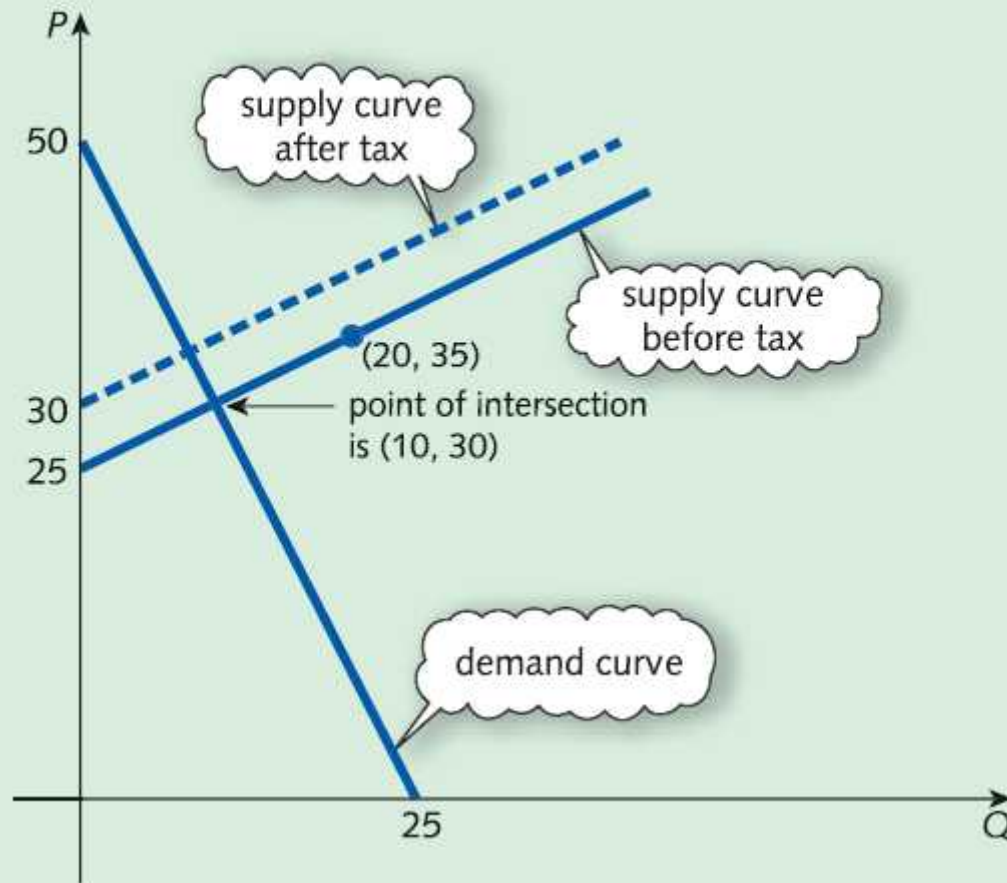
$$P = \frac{1}{2}(20) + 25 = 35$$

so the line also passes through $(20, 35)$. The points $(0, 25)$ and $(20, 35)$ can now be plotted and the supply curve sketched. Figure 1.20 shows both the demand and supply curves sketched on the same diagram. The point of intersection has coordinates $(10, 30)$, so the equilibrium quantity is 10 and the equilibrium price is 30.

It is possible to calculate these values using algebra. In equilibrium, $Q_D = Q_S$. If this common value is denoted by Q then the demand and supply equations become

$$P = -2Q + 50 \quad \text{and} \quad P = \frac{1}{2}Q + 25$$

Figure 1.20



Penyelesaian Secara Simultan

25

$$-2Q + 50 = \frac{1}{2}Q + 25$$

since both sides are equal to P . This can be rearranged to calculate Q :

$$-2\frac{1}{2}Q + 50 = 25 \quad (\text{subtract } \frac{1}{2}Q \text{ from both sides})$$

$$-2\frac{1}{2}Q = -25 \quad (\text{subtract } 50 \text{ from both sides})$$

$$Q = 10 \quad (\text{divide both sides by } -2\frac{1}{2})$$

Finally, P can be found by substituting this value into either of the original equations.

The demand equation gives

$$P = -2(10) + 50 = 30$$

As a check, the supply equation gives

$$P = \frac{1}{2}(10) + 25 = 30 \quad \checkmark$$

Jika ada tambahan pajak?

26

$$P - 5 = \frac{1}{2}Q_S + 25$$

that is,

$$P = \frac{1}{2}Q_S + 30$$

The remaining calculations proceed as before. In equilibrium, $Q_D = Q_S$. Again setting this common value to be Q gives

$$P = -2Q + 50$$

$$P = \frac{1}{2}Q + 30$$

Hence

$$-2Q + 50 = \frac{1}{2}Q + 30$$

which can be solved as before to give $Q = 8$. Substitution into either of the above equations gives $P = 34$.

Siapa yang membayar pajak?

27

Graphically, the introduction of tax shifts the supply curve upwards by 5 units. Obviously the demand curve is unaltered. The dashed line in Figure 1.20 shows the new supply curve, from which the new equilibrium quantity is 8 and equilibrium price is 34. Note the effect that government taxation has on the market equilibrium price. This has risen to \$34 and so not all of the tax is passed on to the consumer. The consumer pays an additional \$4 per good. The remaining \$1 of tax must, therefore, be paid by the firm.

Soal Latihan 3

28

Practice Problem

3 The demand and supply functions of a good are given by

$$P = -4Q_D + 120$$

$$P = \frac{1}{3}Q_S + 29$$

where P , Q_D and Q_S denote the price, quantity demanded and quantity supplied respectively.

(a) Calculate the equilibrium price and quantity.

(b) Calculate the new equilibrium price and quantity after the imposition of a fixed tax of \$13 per good. Who pays the tax?

Barang Substitusi dan Komplementer

29

We conclude this section by considering a more realistic model of supply and demand, taking into account substitutable and complementary goods. Let us suppose that there are two goods in related markets, which we call good 1 and good 2. The demand for either good depends on the prices of both good 1 and good 2. If the corresponding demand functions are linear then

$$Q_{D_1} = a_1 + b_1P_1 + c_1P_2$$

$$Q_{D_2} = a_2 + b_2P_1 + c_2P_2$$

Contoh Soal Barang Substitusi dan Komplementer

30

Example

The demand and supply functions for two interdependent commodities are given by

$$Q_{D_1} = 10 - 2P_1 + P_2$$

$$Q_{D_2} = 5 + 2P_1 - 2P_2$$

$$Q_{S_1} = -3 + 2P_1$$

$$Q_{S_2} = -2 + 3P_2$$

where Q_{D_i} , Q_{S_i} and P_i denote the quantity demanded, quantity supplied and price of good i respectively. Determine the equilibrium price and quantity for this two-commodity model.

Solution

In equilibrium, we know that the quantity supplied is equal to the quantity demanded for each good, so that

$$Q_{D_1} = Q_{S_1} \quad \text{and} \quad Q_{D_2} = Q_{S_2}$$

Let us write these respective common values as Q_1 and Q_2 . The demand and supply equations for good 1 then become

$$Q_1 = 10 - 2P_1 + P_2$$

$$Q_1 = -3 + 2P_1$$

Hence

$$10 - 2P_1 + P_2 = -3 + 2P_1$$

since both sides are equal to Q_1 . It makes sense to tidy this equation up a bit by collecting all of the unknowns on the left-hand side and putting the constant terms on to the right-hand side:

$$10 - 4P_1 + P_2 = -3 \quad (\text{subtract } 2P_1 \text{ from both sides})$$

$$-4P_1 + P_2 = -13 \quad (\text{subtract } 10 \text{ from both sides})$$

We can perform a similar process for good 2. The demand and supply equations become

$$Q_2 = 5 + 2P_1 - 2P_2$$

$$Q_2 = -2 + 3P_2$$

because $Q_{D_2} = Q_{S_2} = Q_2$ in equilibrium. Hence

$$5 + 2P_1 - 2P_2 = -2 + 3P_2$$

$$5 + 2P_1 - 5P_2 = -2 \quad (\text{subtract } 3P_2 \text{ from both sides})$$

$$2P_1 - 5P_2 = -7 \quad (\text{subtract } 5 \text{ from both sides})$$

We have therefore shown that the equilibrium prices, P_1 and P_2 , satisfy the simultaneous linear equations

$$-4P_1 + P_2 = -13 \quad (1)$$

$$2P_1 - 5P_2 = -7 \quad (2)$$

which can be solved by elimination. Following the steps described in Section 1.2 we proceed as follows.

Step 1

Double equation (2) and add to equation (1) to get

$$\begin{array}{r} -4P_1 + P_2 = -13 \\ 4P_1 - 10P_2 = -14 + \\ \hline -9P_2 = -27 \end{array} \quad (3)$$

Step 2

Divide both sides of equation (3) by -9 to get $P_2 = 3$.

Step 3

If this is substituted into equation (1) then

$$\begin{array}{r} -4P_1 + 3 = -13 \\ -4P_1 = -16 \quad (\text{subtract 3 from both sides}) \\ P_1 = 4 \quad (\text{divide both sides by } -4) \end{array}$$

Step 4

As a check, equation (2) gives

$$2(4) - 5(3) = -7 \quad \checkmark$$

Hence $P_1 = 4$ and $P_2 = 3$.

Finally, the equilibrium quantities can be deduced by substituting these values back into the original supply equations. For good 1,

$$Q_1 = -3 + 2P_1 = -3 + 2(4) = 5$$

For good 2,

$$Q_2 = -2 + 3P_2 = -2 + 3(3) = 7$$

As a check, the demand equations also give

$$Q_1 = 10 - 2P_1 + P_2 = 10 - 2(4) + 3 = 5 \quad \checkmark$$

$$Q_2 = 5 + 2P_1 - 2P_2 = 5 + 2(4) - 2(3) = 7 \quad \checkmark$$

Latihan Soal

35

Practice Problem

- 4 The demand and supply functions for two interdependent commodities are given by

$$Q_{D_1} = 40 - 5P_1 - P_2$$

$$Q_{D_2} = 50 - 2P_1 - 4P_2$$

$$Q_{S_1} = -3 + 4P_1$$

$$Q_{S_2} = -7 + 3P_2$$

where Q_{D_i} , Q_{S_i} and P_i denote the quantity demanded, quantity supplied and price of good i respectively. Determine the equilibrium price and quantity for this two-commodity model. Are these goods substitutable or complementary?

36

STOP.....!