PERSAMAAN LINEAR

BAGIAN 2

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Matematika Ekonomi

Algebraic solution of simultaneous linear equations…

At the end of this section you should be able to:

- Solve a system of two simultaneous linear equations in two unknowns using elimination.
- Detect when a system of equations does not have a solution.
- Detect when a system of equations has infinitely many solutions.
- Solve a system of three simultaneous linear equations in three unknowns using elimination.

Kelemahan Metode Grafik

- Skala pengukuran yang rumit
- Penentuan nilai axis y maupun x yang bentuknya angka desimal
- Banyaknya persamaan dalam ekonomi (bisa lebih dari dua, tiga, bahkan empat)

Elimination method

$$4x + 3y = 11 \qquad (1)$$

$$2x + \quad y = 5 \qquad (2)$$

$$4x + 2y = 10 \qquad (3)$$

We may now subtract equation (3) from (1) to get

$$y = 1$$

$$\begin{array}{r} 4x + 3y = 11 \\ 4x + 2y = 10 \; - \\ \hline y = \;\; 1 \end{array}$$

the x's cancel
when you subtract

Lanjut

$$4x + 3(1) = 11 \quad \text{(substitute } y = 1\text{)}$$

$$4x + 3 = 11$$

$$4x = 8 \quad \text{(subtract 3 from both sides)}$$

$$x = 2 \quad \text{(divide both sides by 4)}$$

Hence the solution is $x = 2$, $y = 1$. As a check, substitution of these values into the other original equation (2) gives

$$2(2) + 1 = 5 \quad \checkmark$$

The method of elimination can be summarized as follows

Step 1

Add/subtract a multiple of one equation to/from a multiple of the other to eliminate x.

Step 2

Solve the resulting equation for y.

Step 3

Substitute the value of y into one of the original equations to deduce x.

Step 4

Check that no mistakes have been made by substituting both x and y into the other original equation.

Contoh Soal (1)

Example

Solve the system of equations

$$3x + 2y = 1$$

$$-2x + y = 2$$

Solution

Step 1

The coefficients of x in equations (1) and (2) are 3 and −2 respectively. We can arrange for these to be the same size (but of opposite sign) by multiplying equation (1) by 2 and multiplying (2) by 3. The new equations will then have x coefficients of 6 and −6, so we can eliminate x this time by adding the equations together. The details are as follows.

Doubling the first equation produces

$$6x + 4y = 2 \qquad (3)$$

Tripling the second equation produces

$$-6x + 3y = 6 \qquad (4)$$

If equation (4) is added to equation (3) then

$$\begin{array}{r} 6x + 4y = 2 \\ -6x + 3y = 6 + \\ \hline 7y = 8 \end{array} \qquad (5)$$

the x's cancel when you add

Step 2

Equation (5) can be solved b

$y = 8/7$

Step 3

If 8/7 is substituted for y in equation (1) then

$$3x + 2\left(\frac{8}{7}\right) = 1$$

$$3x + \frac{16}{7} = 1$$

$$3x = 1 - \frac{16}{7} \quad \text{(subtract 16/7 from both sides)}$$

$$3x = \frac{7 - 16}{7} \quad \text{(put over a common denominator)}$$

$$3x = -\frac{9}{7}$$

$$x = \frac{1}{3} \times \left(-\frac{9}{7}\right) \quad \text{(divide both sides by 3)}$$

$$x = -\frac{3}{7}$$

The solution is therefore $x = -3/7$, $y = 8/7$.

Step 4

As a check, equation (2) gives

$$-2\left(-\frac{3}{7}\right)+\frac{8}{7}=\frac{6}{7}+\frac{8}{7}=\frac{6+8}{7}=\frac{14}{7}=2 \quad \checkmark$$

It's easy..!

Practice Problem

1 **(a)** Solve the equations

$$3x - 2y = 4$$
$$x - 2y = 2$$

by eliminating one of the variables.

(b) Solve the equations

$$3x + 5y = 19$$
$$-5x + 2y = -11$$

by eliminating one of the variables.

Contoh Soal (2)

Example

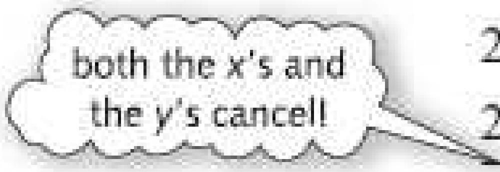
Solve the system of equations

$$x - 2y = 1$$
$$2x - 4y = -3$$

Solution

Step 1

The variable x can be eliminated by doubling the first equation and subtracting the second:



$$\begin{array}{r} 2x - 4y = 2 \\ 2x - 4y = -3 \; - \\ \hline 0 = 5 \end{array}$$

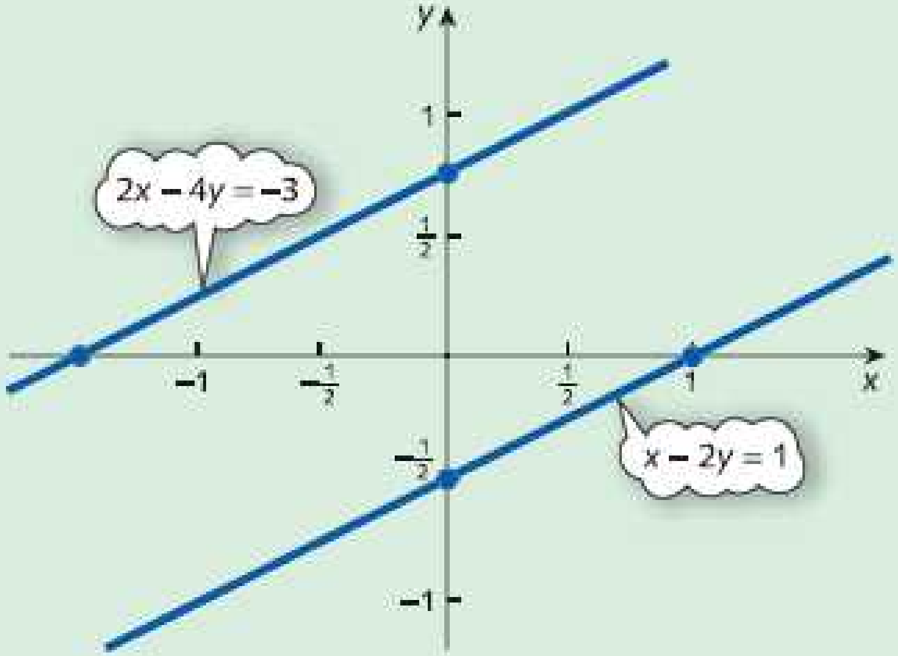
The line $x - 2y = 1$ passes through the points $(0, -1/2)$ and $(1, 0)$ (check this). The line $2x - 4y = -3$ passes through the points $(0, 3/4)$ and $(-3/2, 0)$ (check this). Figure 1.11 shows that these lines are parallel and so they do not intersect. It is therefore not surprising that we were unable to find a solution using algebra, because this system of equations does not have one. We could have deduced this before when subtracting the equations. The equation that only involves y in step 2 can be written as

$$0y = 5$$

and the problem is to find a value of y for which this equation is true. No such value exists, since

$$\boxed{\text{zero}} \times \boxed{\text{any number}} = \boxed{\text{zero}}$$

Figure 1.11



Contoh Soal (3)

Example

Solve the equations

$$2x - 4y = 1$$

$$5x - 10y = 5/2$$

Solution

Step 1

The variable x can be eliminated by multiplying the first equation by 5, multiplying the second equation by 2 and subtracting

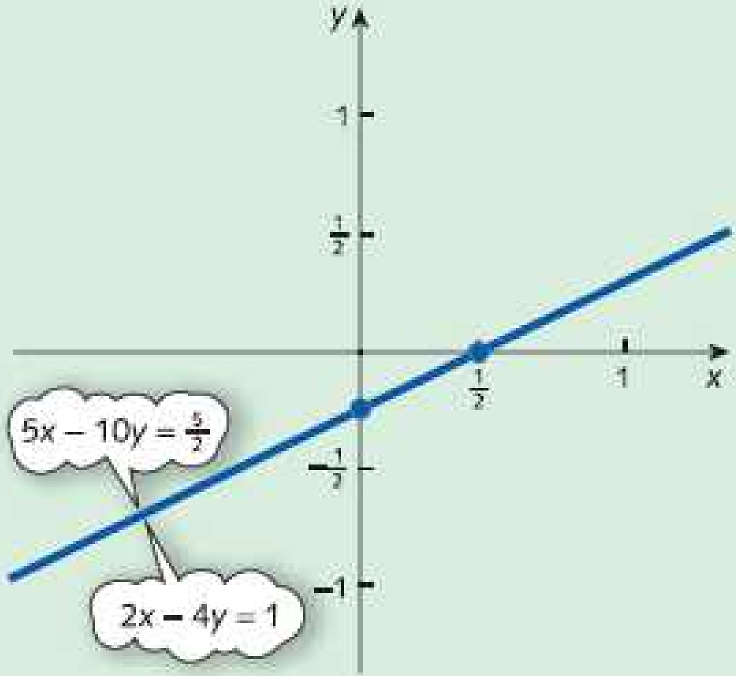
$$\begin{array}{r} 10x - 20y = 5 \\ \underline{10x - 20y = 5} - \\ 0 = 0 \end{array}$$

everything cancels including the right-hand side!

Again, it is easy to explain this using graphs. The line $2x - 4y = 1$ passes through $(0, -1/4)$ and $(1/2, 0)$. The line $5x - 10y = 5/2$ passes through $(0, -1/4)$ and $(1/2, 0)$. Consequently, both equations represent the same line. From Figure 1.12 the lines intersect along the whole of their length and any point on this line is a solution. This particular system of equations has infinitely many solutions. This can also be deduced algebraically. The equation involving y in step 2 is

$$0y = 0$$

Figure 1.12



Practice Problem

2 Attempt to solve the following systems of equations

(a) $3x - 6y = -2$
$-4x + 8y = -1$

(b) $-5x + y = 4$
$10x - 2y = -8$

Comment on the nature of the solution in each case.

Ada pertanyaan?

3 Persamaan Linear

$$x + 3y - z = 4 \quad (1)$$

$$2x + y + 2z = 10 \quad (2)$$

$$3x - y + z = 4 \quad (3)$$

Berapa nilai x, y, dan z?

The variable x can be eliminated from the second equation by multiplying equation (1) by 2 and subtracting equation (2):

$$\begin{array}{r} 2x + 6y - 2z = 8 \\ 2x + y + 2z = 10 - \\ \hline 5y - 4z = -2 \end{array} \qquad (4)$$

Similarly, we can eliminate x from the third equation by multiplying equation (1) by 3 and subtracting equation (3):

$$\begin{array}{r} 3x + 9y - 3z = 12 \\ 3x - y + z = 4 - \\ \hline 10y - 4z = 8 \end{array} \qquad (5)$$

At this stage the first equation is unaltered but the second and third equations of the system have changed to equations (4) and (5) respectively, so the current equations are

$$x + 3y - z = 4 \qquad (1)$$

$$5y - 4z = -2 \qquad (4)$$

$$10y - 4z = 8 \qquad (5)$$

We can eliminate y in the last equation by multiplying equation (4) by 2 and subtracting equation (5):

$$\begin{array}{r} 10y - 8z = -4 \\ \underline{10y - 4z = 8} - \\ -4z = -12 \end{array} \qquad (6)$$

Collecting together the current equations gives

$$x + 3y - z = 4 \qquad (1)$$

$$5y - 4z = -2 \qquad (4)$$

$$-4z = -12 \qquad (6)$$

From the last equation,

$$z = \frac{-12}{-4} = 3 \qquad \text{(divide both sides by } -4\text{)}$$

If this is substituted into equation (4) then

$$5y - 4(3) = -2$$

$$5y - 12 = -2$$

$$5y = 10 \quad \text{(add 12 to both sides)}$$

$$y = 2 \quad \text{(divide both sides by 5)}$$

Finally, substituting $y = 2$ and $z = 3$ into equation (1) produces

$$x + 3(2) - 3 = 4$$

$$x + 3 = 4$$

$$x = 1 \quad \text{(subtract 3 from both sides)}$$

Hence the solution is $x = 1$, $y = 2$, $z = 3$.

As usual, it is possible to check the answer by putting these numbers back into the original equations (1), (2) and (3)

$$1 + 3(2) - 3 = 4 \quad ✓$$

$$2(1) + 2 + 2(3) = 10 \quad ✓$$

$$3(1) - 2 + 3 = 4 \quad ✓$$

The general strategy may be summarized as follows. Consider the system

$$?x + ?y + ?z = ?$$

$$?x + ?y + ?z = ?$$

$$?x + ?y + ?z = ?$$

where ? denotes some numerical coefficient.

BAGAIMANA STRATEGINYA?

Step 1

Add/subtract multiples of the first equation to/from multiples of the second and third equations to eliminate x. This produces a new system of the form

$$\begin{aligned} ?x + ?y + ?z &= ? \\ ?y + ?z &= ? \\ ?y + ?z &= ? \end{aligned}$$

Step 2

Add/subtract a multiple of the second equation to/from a multiple of the third to eliminate y. This produces a new system of the form

$$\begin{aligned} ?x + ?y + ?z &= ? \\ ?y + ?z &= ? \\ ?z &= ? \end{aligned}$$

Step 3

Solve the last equation for z. Substitute the value of z into the second equation to deduce y. Finally, substitute the values of both y and z into the first equation to deduce x.

Step 4

Check that no mistakes have been made by substituting the values of x, y and z into the original equations.

Contoh Soal (4)

Example

Solve the equations

$$4x + y + 3z = 8 \quad (1)$$

$$-2x + 5y + z = 4 \quad (2)$$

$$3x + 2y + 4z = 9 \quad (3)$$

Solution

Step 1

To eliminate x from the second equation we multiply it by 2 and add to equation (1):

$$\begin{array}{r} 4x + y + 3z = 8 \\ -4x + 10y + 2z = 8 \; + \\ \hline 11y + 5z = 16 \end{array} \quad (4)$$

To eliminate x from the third equation we multiply equation (1) by 3, multiply equation (3) by 4 and subtract:

$$\begin{array}{r} 12x + 3y + 9z = 24 \\ 12x + 8y + 16z = 36 \;- \\ \hline -5y - 7z = -12 \end{array} \qquad (5)$$

This produces a new system:

$$4x + y + 3z = 8 \qquad (1)$$

$$11y + 5z = 16 \qquad (4)$$

$$-5y - 7z = -12 \qquad (5)$$

Step 2

To eliminate y from the new third equation (that is, equation (5)) we multiply equation (4) by 5, multiply equation (5) by 11 and add:

$$\begin{array}{rcrl} 55y + 25z & = & 80 & \\ -55y - 77z & = & -132 & + \\ \hline -52z & = & -52 & \end{array} \qquad (6)$$

This produces a new system

$$4x + y + 3z = 8 \qquad (1)$$

$$11y + 5z = 16 \qquad (4)$$

$$-52z = -52 \qquad (6)$$

Step 3

The last equation gives

$$z = \frac{-52}{-52} = 1 \quad \text{(divide both sides by } -52\text{)}$$

If this is substituted into equation (4) then

$$11y + 5(1) = 16$$

$$11y + 5 = 16$$

$$11y = 11 \quad \text{(subtract 5 from both sides)}$$

$$y = 1 \quad \text{(divide both sides by 11)}$$

Finally, substituting $y = 1$ and $z = 1$ into equation (1) produces

$$4x + 1 + 3(1) = 8$$

$$4x + 4 = 8$$

$$4x = 4 \quad \text{(subtract 4 from both sides)}$$

$$x = 1 \quad \text{(divide both sides by 4)}$$

Hence the solution is $x = 1$, $y = 1$, $z = 1$.

Step 4

As a check the original equations (1), (2) and (3) give

$$4(1) + 1 + 3(1) = 8 \quad \checkmark$$

$$-2(1) + 5(1) + 1 = 4 \quad \checkmark$$

$$3(1) + 2(1) + 4(1) = 9 \quad \checkmark$$

respectively.

Practice Problem

3 Solve the following system of equations:

$$2x + 2y - 5z = -5 \quad (1)$$

$$x - y + z = 3 \quad (2)$$

$$-3x + y + 2z = -2 \quad (3)$$

Stop..!