Aplikasi Diferensiasi: Elastisitas

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Introduction

- One important problem in business is to determine the effect on revenue of a change in the price of a good.
- If the firm lowers the price then it will receive less for each item, but the number of items sold increases.
- The crucial factor here is not the absolute changes in *P and Q but rather the proportional* or percentage changes.

Types of Elasticity

- If the percentage rise in Q is greater than the percentage fall in P then the firm experiences an increase in revenue. Under these circumstances we say that demand is **elastic.**
- Demand is said to be **inelastic** if demand is relatively insensitive to price changes. In this case, the percentage change in quantity is less than the percentage change in price.
- If the percentage changes in price and quantity are equal, leaving revenue unchanged. We use the term **unit elastic** to describe this situation.

Price elasticity of demand

$$E = \frac{\text{percentage change in demand}}{\text{percentage change in price}}$$

Notice that because the demand curve slopes downwards, a positive change in price leads to a negative change in quantity and vice versa. Consequently, the value of *E* is always negative. It is conventional to avoid this by deliberately changing the sign and taking

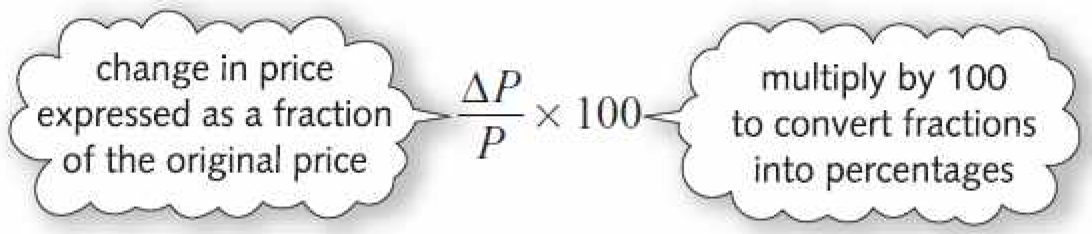
$$E = -\frac{\text{percentage change in demand}}{\text{percentage change in price}}$$

which makes *E* positive. The previous classification of demand functions can now be restated more succinctly in terms of *E*.

Demand is said to be

- inelastic if $E < 1$
- unit elastic if $E = 1$
- elastic if $E > 1$.

Rumus Elastisitas



$$\frac{\Delta P}{P} \times 100$$

Similarly, the percentage change in quantity is

$$\frac{\Delta Q}{Q} \times 100$$

Next

Hence

$$E = -\left(\frac{\Delta Q}{Q} \times 100\right) \div \left(\frac{\Delta P}{P} \times 100\right)$$

Now, when we divide two fractions we turn the denominator upside down and multiply, so

$$E = -\left(\frac{\Delta Q}{Q} \times \cancel{100}\right) \times \left(\frac{P}{\cancel{100} \times \Delta P}\right)$$

$$= -\frac{P}{Q} \times \frac{\Delta Q}{\Delta P}$$

Example

Example

Determine the elasticity of demand when the price falls from 136 to 119, given the demand function

$$P = 200 - Q^2$$

Solution

In the notation of Figure 4.19 we are given that

$$P_1 = 136 \quad \text{and} \quad P_2 = 119$$

The corresponding values of Q_1 and Q_2 are obtained from the demand equation

$$P = 200 - Q^2$$

by substituting $P = 136$ and 119 respectively and solving for Q. For example, if $P = 136$ then

$$136 = 200 - Q^2$$

which rearranges to give

$$Q^2 = 200 - 136 = 64$$

This has solution $Q = \pm 8$ and, since we can obviously ignore the negative quantity, we have $Q_1 = 8$. Similarly, setting $P = 119$ gives $Q_2 = 9$. The elasticity formula is

$$E = -\frac{P}{Q} \times \frac{\Delta Q}{\Delta P}$$

and the values of ΔP and ΔQ are easily worked out to be

$$\Delta P = 119 - 136 = -17$$

$$\Delta Q = 9 - 8 = 1$$

However, it is not at all clear what to take for P and Q. Do we take P to be 136 or 119? Clearly we are going to get two different answers depending on our choice. A sensible compromise is to use their average and take

$$P = {}^1\!/\!_2(136 + 119) = 127.5$$

Similarly, averaging the Q values gives

$$Q = {}^1\!/\!_2(8 + 9) = 8.5$$

Hence

$$E = -\frac{127.5}{8.5} \times \left(\frac{1}{-17}\right) = 0.88$$

Practice Problem

1 Given the demand function

$$P = 1000 - 2Q$$

calculate the arc elasticity as P falls from 210 to 200.

Example

Given the demand function

$$P = 50 - 2Q$$

find the elasticity when the price is 30. Is demand inelastic, unit elastic or elastic at this price?

Solution

To find dQ/dP we need to differentiate Q with respect to P. However, we are actually given a formula for P in terms of Q, so we need to transpose

$$P = 50 - 2Q$$

for Q. Adding $2Q$ to both sides gives

$$P + 2Q = 50$$

and if we subtract P then

$$2Q = 50 - P$$

Finally, dividing through by 2 gives

$$Q = 25 - \tfrac{1}{2}P$$

Hence

$$\frac{dQ}{dP} = -\tfrac{1}{2}$$

We are given that $P = 30$ so, at this price, demand is

$$Q = 25 - \tfrac{1}{2}(30) = 10$$

These values can now be substituted into

$$E = -\frac{P}{Q} \times \frac{\mathrm{d}Q}{\mathrm{d}P}$$

to get

$$E = -\frac{30}{10} \times \left(-\frac{1}{2}\right) = 1.5$$

Moreover, since $1.5 > 1$, demand is elastic at this price.

Practice Problem

2 Given the demand function

$$P = 100 - Q$$

calculate the price elasticity of demand when the price is

(a) 10 **(b)** 50 **(c)** 90

Is the demand inelastic, unit elastic or elastic at these prices?

Example

Given the demand function

$$P = -Q^2 - 4Q + 96$$

find the price elasticity of demand when $P = 51$. If this price rises by 2%, calculate the corresponding percentage change in demand.

Solution

We are given that $P = 51$, so to find the corresponding demand we need to solve the quadratic equation

$$-Q^2 - 4Q + 96 = 51$$

that is,

$$-Q^2 - 4Q + 45 = 0$$

To do this we use the standard formula

$$\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

discussed in Section 2.1, which gives

$$Q = \frac{-(-4) \pm \sqrt{((-4)^2 - 4(-1)(45))}}{2(-1)}$$

$$= \frac{4 \pm \sqrt{196}}{-2}$$

$$= \frac{4 \pm 14}{-2}$$

The two solutions are −9 and 5. As usual, the negative value can be ignored, since it does not make sense to have a negative quantity, so $Q = 5$.

To find the value of E we also need to calculate

$$\frac{dQ}{dP}$$

from the demand equation, $P = -Q^2 - 4Q + 96$. It is not at all easy to transpose this for Q. Indeed, we would have to use the formula for solving a quadratic, as above, replacing the number 51 by the letter P. Unfortunately this expression involves square roots and the subsequent differentiation is quite messy. (You might like to have a go at this yourself!) However, it is easy to differentiate the given expression with respect to Q to get

$$\frac{dP}{dQ} = -2Q - 4$$

and so

$$\frac{dQ}{dP} = \frac{1}{dP/dQ} = \frac{1}{-2Q - 4}$$

Finally, putting $Q = 5$ gives

$$\frac{dQ}{dP} = -\frac{1}{14}$$

The price elasticity of demand is given by

$$E = -\frac{P}{Q} \times \frac{dQ}{dP}$$

and if we substitute $P = 51$, $Q = 5$ and $dQ/dP = -1/14$ we get

$$E = -\frac{51}{5} \times \left(-\frac{1}{14}\right) = 0.73$$

To discover the effect on Q due to a 2% rise in P we return to the original definition

$$E = -\frac{\text{percentage change in demand}}{\text{percentage change in price}}$$

We know that $E = 0.73$ and that the percentage change in price is 2, so

$$0.73 = -\frac{\text{percentage change in demand}}{2}$$

which shows that demand changes by

$$-0.73 \times 2 = -1.46\%$$

A 2% rise in price therefore leads to a fall in demand of 1.46%.

Practice Problem

3 Given the demand equation

$$P = -Q^2 - 10Q + 150$$

find the price elasticity of demand when Q − 4. Estimate the percentage change in price needed to increase demand by 10%.

Price elasticity of supply

The *price elasticity of supply* is defined in an analogous way to that of demand. We define

$$E = \frac{\text{percentage change in supply}}{\text{percentage change in price}}$$

This time, however, there is no need to fiddle the sign. An increase in price leads to an increase in supply, so E is automatically positive. In symbols,

$$E = \frac{P}{Q} \times \frac{\Delta Q}{\Delta P}$$

Example

Example

Given the supply function

$$P = 10 + \sqrt{Q}$$

find the price elasticity of supply

(**a**) averaged along an arc between $Q = 100$ and $Q = 105$

(**b**) at the point $Q = 100$

Solution

(a) We are given that

$$Q_1 = 100,\ Q_2 = 105$$

so that

$$P_1 = 10 + \sqrt{100} = 20 \quad \text{and} \quad P_2 = 10 + \sqrt{105} = 20.247$$

Hence

$$\Delta P = 20.247 - 20 = 0.247, \qquad \Delta Q = 105 - 100 = 5$$

$$P = \frac{1}{2}(20 + 20.247) = 20.123, \qquad Q = \frac{1}{2}(100 + 105) = 102.5$$

The formula for arc elasticity gives

$$E = \frac{P}{Q} \times \frac{\Delta Q}{\Delta P} = \frac{20.123}{102.5} \times \frac{5}{0.247} = 3.97$$

(b) To evaluate the elasticity at the point $Q = 100$, we need to find the derivative, $\frac{dQ}{dP}$. The supply equation

$$P = 10 + Q^{1/2}$$

differentiates to give

$$\frac{dP}{dQ} = \frac{1}{2}Q^{-1/2} = \frac{1}{2\sqrt{Q}}$$

so that

$$\frac{dQ}{dP} = 2\sqrt{Q}$$

At the point $Q = 100$, we get

$$\frac{dQ}{dP} = 2\sqrt{100} = 20$$

The formula for point elasticity gives

$$E = \frac{P}{Q} \times \frac{dQ}{dP} = \frac{20}{100} \times 20 = 4$$

Notice that, as expected, the answers to parts (a) and (b) are nearly the same.

Latihan Soal

1. Find the price elasticity of demand at P = 6 for each of the following demand functions

(a) $P = 30 - 2Q$

(b) $P = 30 - 12Q$

(c) $P = \sqrt{(100 - 2Q)}$

Latihan Soal (2)

2. Consider the supply equation

$$Q = 4 + 0.1P^2$$

(a) Write down an expression for dQ/dP.

(b) Show that the supply equation can be rearranged as

$$P = \sqrt{(10Q - 40)}$$

Differentiate this to find an expression for dP/dQ.

(c) Use your answers to parts (a) and (b) to verify that

$$\frac{dQ}{dP} = \frac{1}{dP/dQ}$$

(d) Calculate the elasticity of supply at the point $Q = 14$.

Stop!!

- Minggu depan bisa dibaca:

"Optimization of economic functions"