



Diferensiasi



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The derivative of a function

- Find the slope of a straight line given any two points on the line.
- Detect whether a line is uphill, downhill or horizontal using the sign of the slope.
- Recognize the notation $f'(x)$ and dy/dx for the derivative of a function.
- Estimate the derivative of a function by measuring the slope of a tangent.
- Differentiate power functions.

Slope



Slope



Garis lurus
/ linear



Garis
melengkung
/ non linear

Contoh Soal Slope Garis Lurus

∞ Find the slope of the straight line passing through:

(a) A (1, 2) and B (3, 4)

(b) A (1, 2) and C (4, 1)

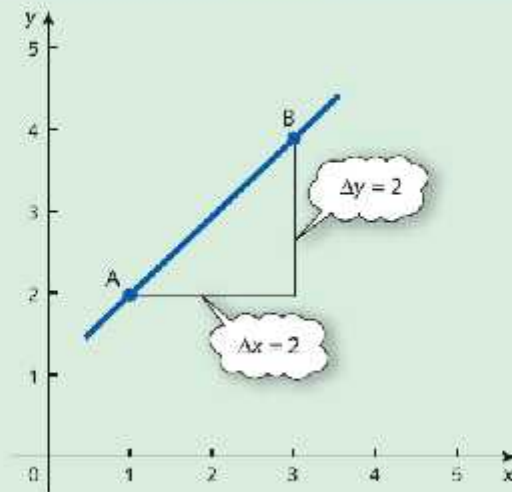
(c) A (1, 2) and D (5, 2)

Jawab

- (a) Points A and B are sketched in Figure 4.1. As we move from A to B, the y coordinate changes from 2 to 4, which is an increase of 2 units, and the x coordinate changes from 1 to 3, which is also an increase of 2 units. Hence

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{4 - 2}{3 - 1} = \frac{2}{2} = 1$$

Figure 4.1

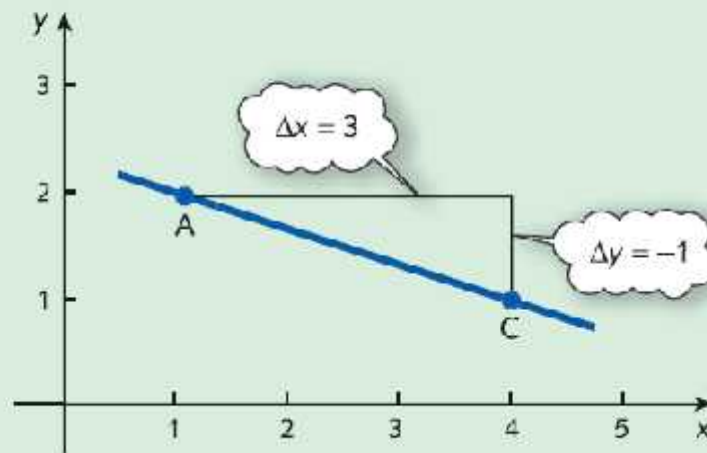


Jawab (2)

- (b) Points A and C are sketched in Figure 4.2. As we move from A to C, the y coordinate changes from 2 to 1, which is a decrease of 1 unit, and the x coordinate changes from 1 to 4, which is an increase of 3 units. Hence

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{1 - 2}{4 - 1} = \frac{-1}{3}$$

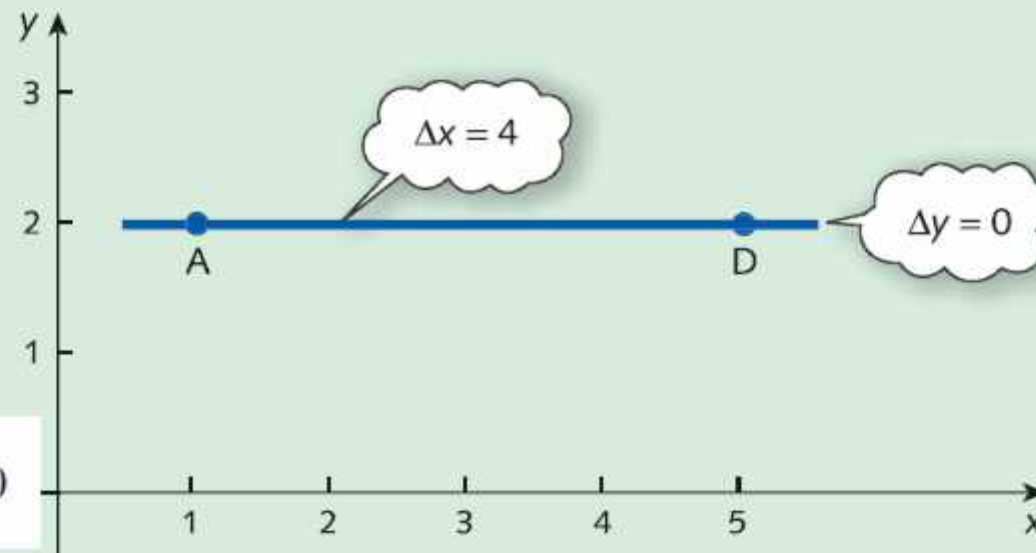
Figure 4.2



Jawab (3)

- (c) Points A and D are sketched in Figure 4.3. As we move from A to D, the y coordinate remains fixed at 2, and the x coordinate changes from 1 to 5, which is an increase of 4 units. Hence

Figure 4.3



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2 - 2}{5 - 1} = \frac{0}{4} = 0$$

Soal Latihan (1)

∞ Find the slope of the straight line passing through:

(a) E $(-1, 3)$ and F $(3, 11)$

(b) E $(-1, 3)$ and G $(4, -2)$

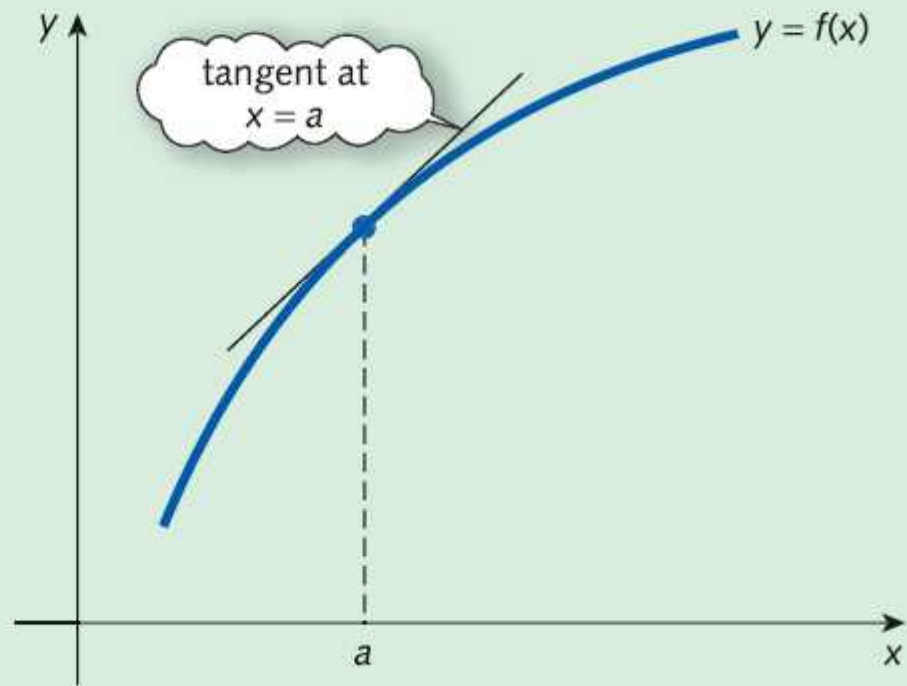
(c) E $(-1, 3)$ and H $(49, 3)$

Catatan

- From these examples we see that the gradient is positive if the line is uphill, negative if the line is downhill and zero if the line is horizontal.
- Unfortunately, not all functions in economics are linear, so it is necessary to extend the definition of slope to include more general curves. To do this we need the idea of a tangent, which is illustrated in Figure 4.4.
- A straight line which passes through a point on a curve and which just touches the curve at this point is called a *tangent*

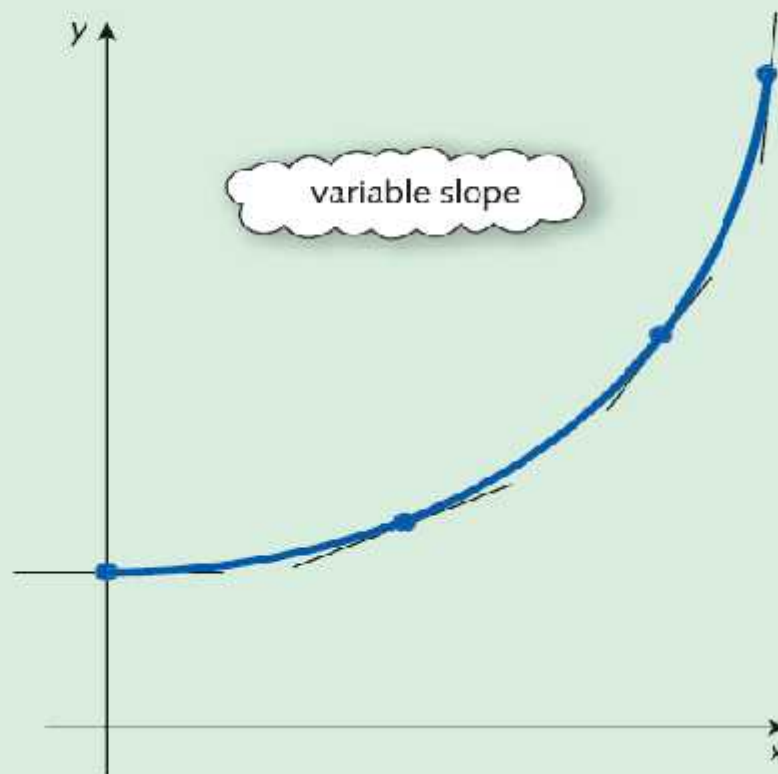
Tangent

Figure 4.4



Tangent (2)

Figure 4.5



Slope garis lurus vs melengkung

☞ Slope garis lurus:

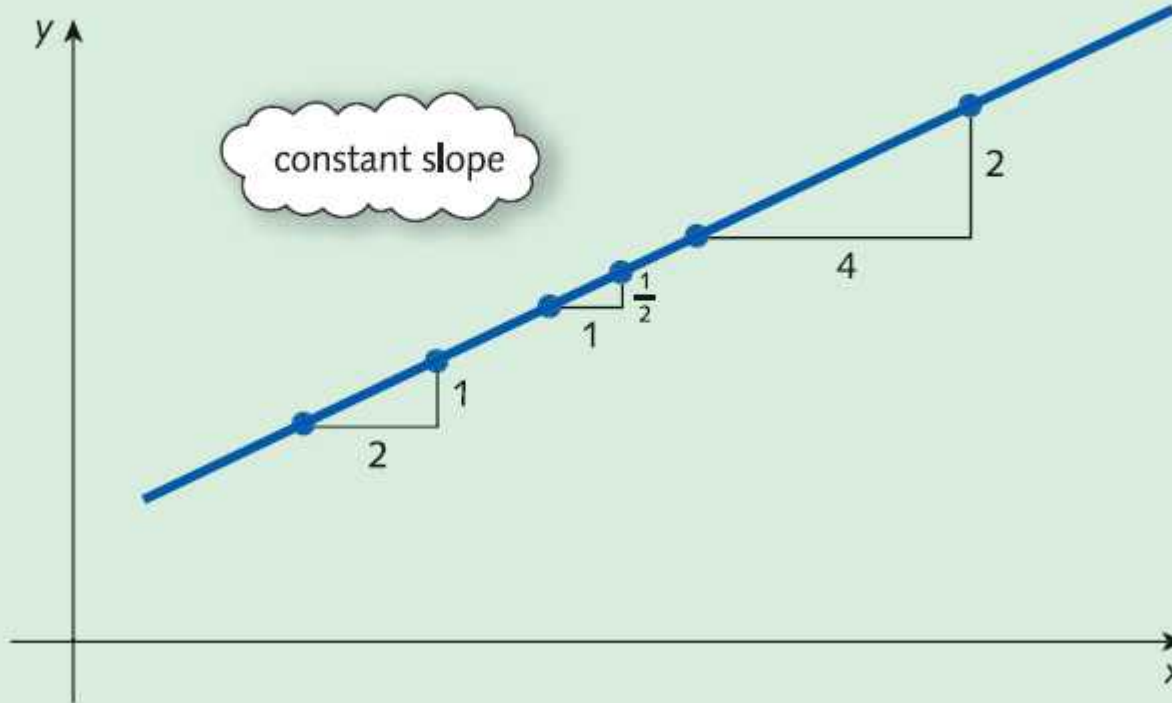
- Memiliki kemiringan yang tetap

☞ Slope garis melengkung:

- Memiliki kemiringan yang berubah-ubah

Slope Garis Lurus

Figure 4.6



Slope Garis Melengkung

∞ However, as we have just seen, the slope of a curve varies as we move along it. In mathematics we use the symbol

$f'(a)$

read 'f dashed of a'

Slope Garis Melengkung (2)

The slope of the graph of a function is called the *derivative* of the function. It is interesting to notice that corresponding to each value of x there is a uniquely defined derivative $f'(x)$. In other words, the rule 'find the slope of the graph of f at x ' defines a function. This slope function is usually referred to as the *derived function*. An alternative notation for the derived function is

$$\frac{dy}{dx}$$

read 'dee y by dee x'

- ⌘ However, it is important to realize that dy/dx does not mean 'dy divided by dx'. It should be thought of as a single symbol representing the derivative of y with respect to x .

Contoh Soal Slope Garis Melengkung

Complete the following table of function values and hence sketch an accurate graph of $f(x) = x^2$.

x	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0
$f(x)$									

Draw the tangents to the graph at $x = -1.5, -0.5, 0, 0.5$ and 1.5 . Hence estimate the values of $f'(-1.5)$, $f'(-0.5)$, $f'(0)$, $f'(0.5)$ and $f'(1.5)$.

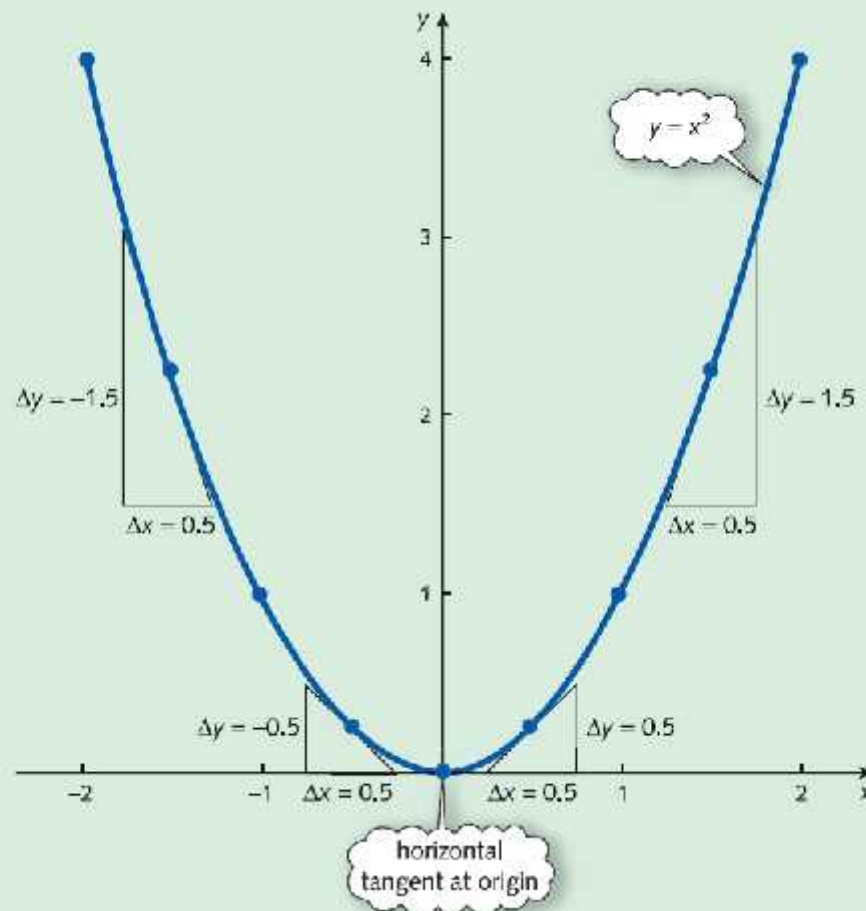
Jawab

Using a calculator we obtain

x	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0
$f(x)$	4	2.25	1	0.25	0	0.25	1	2.25	4

Grafiknya

Figure 4.7



Slopenya

∞ The corresponding graph of the square function is sketched in Figure 4.7. From the graph we see that the slopes of the tangents are

$$f'(-1.5) = \frac{-1.5}{0.5} = -3$$

$$f'(-0.5) = \frac{-1.5}{0.5} = -1$$

$$f'(0) = 0$$

$$f'(0.5) = \frac{0.5}{0.5} = 1$$

$$f'(1.5) = \frac{1.5}{0.5} = 3$$

Contoh Soal Slope Garis Melengkung (2)

Complete the following table of function values and hence sketch an accurate graph of $f(x) = x^3$.

x	-1.50	-1.25	-1.00	-0.75	-0.50	-0.25	0.00
$f(x)$		-1.95			-0.13		
x	0.25	0.50	0.75	1.00	1.25	1.50	
$f(x)$		0.13			1.95		

Draw the tangents to the graph at $x = -1, 0$ and 1 . Hence estimate the values of $f'(-1)$, $f'(0)$ and $f'(1)$.

Rumus

$$\text{if } f(x) = x^n \text{ then } f'(x) = nx^{n-1}$$

or, equivalently,

$$\text{if } y = x^n \text{ then } \frac{dy}{dx} = nx^{n-1}$$

Cara Menderivasi

The process of finding the derived function symbolically (rather than using graphs) is known as *differentiation*. In order to differentiate x^n all that needs to be done is to bring the power down to the front and then to subtract 1 from the power:

x^n differentiates to nx^{n-1}

bring down
the power

subtract 1 from
the power

Cara Menderivasi (2)

To differentiate the square function we set $n = 2$ in this formula to deduce that

$$f(x) = x^2 \text{ differentiates to } f'(x) = 2x^{2-1}$$

subtract 1

the 2 comes down

that is,

$$f'(x) = 2x^1 = 2x$$

∞ Lebih lanjut..

$$f'(-1.5) = 2 \times (-1.5) = -3$$

$$f'(-0.5) = 2 \times (-0.5) = -1$$

$$f'(0) = 2 \times (0) = 0$$

$$f'(0.5) = 2 \times (0.5) = 1$$

$$f'(1.5) = 2 \times (1.5) = 3$$

Contoh Soal Derivatif

Differentiate

(a) $y = x^4$

(b) $y = x^{10}$

(c) $y = x$

(d) $y = 1$

(e) $y = 1/x^4$

(f) $y = \sqrt{x}$

Jawab

- (a) To differentiate $y = x^4$ we bring down the power (that is, 4) to the front and then subtract 1 from the power (that is, $4 - 1 = 3$) to deduce that

$$\frac{dy}{dx} = 4x^3$$

- (b) Similarly,

$$\text{if } y = x^{10} \text{ then } \frac{dy}{dx} = 10x^9$$

- (c) To use the general formula to differentiate x we first need to express $y = x$ in the form $y = x^n$ for some number n . In this case $n = 1$ because $x^1 = x$, so

$$\frac{dy}{dx} = 1x^0 = 1 \quad \text{since } x^0 = 1$$

This result is also obvious from the graph of $y = x$ sketched in Figure 4.8.

Jawab (2)

(d) Again, to differentiate 1 we need to express $y = 1$ in the form $y = x^n$. In this case $n = 0$ because $x^0 = 1$, so

$$\frac{dy}{dx} = 0x^{-1} = 0$$

This result is also obvious from the graph of $y = 1$ sketched in Figure 4.9.

(e) Noting that $1/x^4 = x^{-4}$ it follows that

$$\text{if } y = \frac{1}{x^4} \text{ then } \frac{dy}{dx} = -4x^{-5} = -\frac{4}{x^5}$$

The power has decreased to -5 because $-4 - 1 = -5$.

Jawab (3)

(f) Noting that $\sqrt{x} = x^{1/2}$ it follows that if

$$y = \sqrt{x} \quad \text{then} \quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{2x^{1/2}}$$

negative powers denote reciprocals

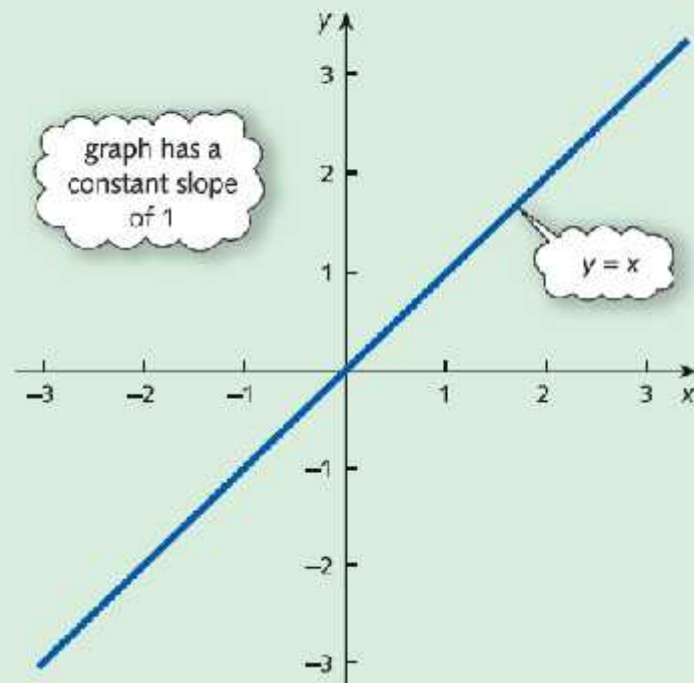
$$= \frac{1}{2\sqrt{x}}$$

fractional powers denote roots

The power has decreased to $-1/2$ because $1/2 - 1 = -1/2$.

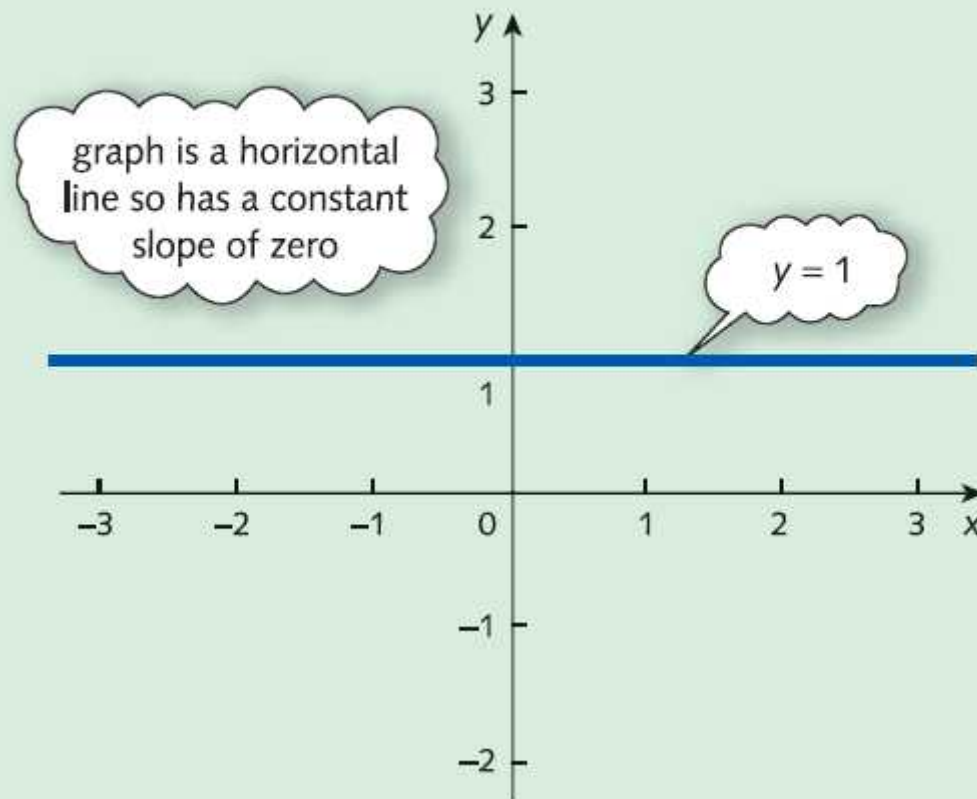
Grafik Soal C

Figure 4.8



Grafik Soal D

Figure 4.9



Latihan Soal

Differentiate

(a) $y = x^5$

(b) $y = x^6$

(c) $y = x^{100}$

(d) $y = 1/x$

(e) $y = 1/x^2$

[Hint: in parts (d) and (e) note that $1/x = x^{-1}$ and $1/x^2 = x^{-2}$.]



Selesai



6/28/2016

al muiz

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Tugas

- ☞ Kerjakan Soal Latihan Halaman 249-250, buku Jacques (2006), untuk Nomor: 5, 9, 10, 12, dan 13
- ☞ Minggu depan dikumpulkan, tulis pada kertas folio garis